Forbidden activation levels in a non-stationary tunneling process

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Tunneling in the presence of an opaque barrier, part of which varies in time, is investigated numerically and analytically in one dimension. Clearly, due to the varying barrier a tunneling particle experiences spectral widening. However, in the case of strong perturbations, the particles' activation to certain energies is avoided. We show that this effect occurs only when the perturbation decays faster than t^{-2}

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Tunneling, which is a very common phenomenon in the quantum world, is rarely a stationary process. In most cases, it is involved with temporal changes of the medium through which the particle is tunneling. In some cases, such variations are mostly due to transport of the tunneling particles themselves. In these cases, the problems are intrinsically nonlinear. However, when the source of the changes is external, the problem can be treated as a linear one. In this category, one can even find stationary many-body transport problems where a single particle can experience the influence of its surroundings (in this case its neighbors) as an alternating external force. Many works (see, for example, refs. [1–5]) done in this field have revealed an abundance of physical effects: resonances and oscillations in energy, exponential increase of the tunneling current, activation-assisted tunneling and elevator resonance activation.

This paper discusses the influence of a very strong perturbation (compared with the incoming particles' energy) on the tunneling process. It is assumed that the perturbation potential is always positive (see Fig.1), i.e., it increases the barrier (locally) and never creates a local well (inside the barrier). Therefore, effects like elevator resonance activation do not occur (see ref. [4]). On the contrary, in the adiabatic approximation, the perturbation - being very strong - should reduce the transmission dramatically. However, when the characteristic time of the perturbation decreases, the particle may absorb energy from the external perturbation, which can assist it in its due course of tunneling. It is clear, for example, that in the extreme case, where the potential barrier is smaller than $\hbar/\tau$ (τ is the characteristic time of the perturbation), the particle can easily (with high probability) absorb enough energy to leap over the barrier. This paper discusses the intermediate case, where the process has a tunneling nature but allows for energy quanta to be absorbed by the tunneling particle. A very strong perturbation, which is activated for a finite period, permits propagation only at the very onset and at the final decay of the perturbation. When the wave function in these two instances has the same phase, maximum transmission is expected, but when in these two moments the wave function is out of phase, one should expect minimum transmission. These are the forbidden activation energies we are looking for.

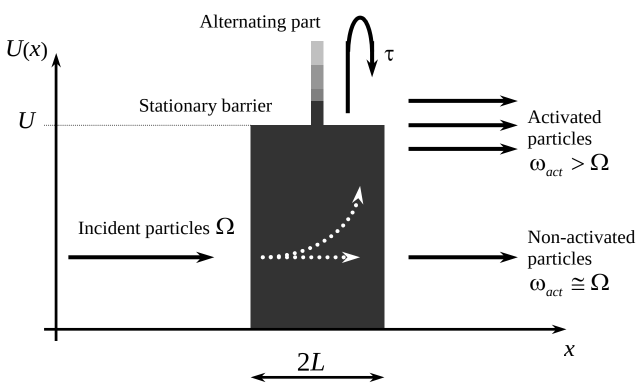

FIG. 1. Activation in a tunneling process due to a varying potential barrier. The different gray levels represent the temporally varying part of the potential.

This system, which is presented in Fig.1, can be described in terms of the Schrödinger equation in the following way

$$-\frac{d^2\psi}{dx^2}+U(x)\psi+f(t/\tau)\delta(x)\psi=i\frac{d\psi}{dt} \qquad (1)$$

where we have used the units $\hbar=1$ and $2m=1$ (m is the electron's mass), and $f(x)$ is a function, which vanishes very rapidly for $x\to\infty$, and $U(x)$ is the barrier's potential.

We seek the solution

$$\psi(x,t)=\psi_\Omega(x,t)+\int d\omega e^{-i\omega t}a_\omega G^+_\omega(x) \qquad (2)$$

where $\psi_\Omega(x,t)$ is the homogeneous solution of eq.1, $\psi_\Omega(x,t)\equiv\varphi_\Omega(x)e^{-i\Omega t}$, while φ solves the stationary-state equation

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$$-\frac{d^2\varphi_\Omega}{dx^2}+\left[U(x)-\Omega\right]\varphi_\Omega=0; \tag{3}$$

G^+_ω is the "outgoing" Green function of equation 3 (but with a general ω instead of Ω).

The perturbation can be regarded as the force that is activated upon the particle for a finite period τ (f has momentum dimensions, therefore f/τ has dimensions of force).

A straightforward substitution of eq.2 in eq.1 reveals the integral equation [4]

$$\tilde{f}(\omega-\Omega)\varphi_\Omega(0)+a_\omega-\int d\omega'\tilde{f}(\omega-\omega')a_{\omega'}G^+_{\omega'}(0)=0 \tag{4}$$

where

$$\tilde{f}(\omega)\equiv(2\pi)^{-1}\int dt f(t/\tau)\exp(i\omega t) \tag{5}$$

and some tedious calculations [6] reveal that, for a rectangular barrier with potential height U and width L,

$$G_\omega(0)=-\frac{\coth\left[\kappa L+i\arctan(k/\kappa)\right]}{2\kappa}\simeq-(2\kappa)^{-1} \tag{6}$$

where $k\equiv\sqrt{\omega}$ and $\kappa\equiv\sqrt{U-\omega}$ and the approximation on the right hand side takes place for an opaque barrier $\left(\sqrt{U}L\gg1\right)$.

It should be noted that the results of this paper are valid for any opaque barrier. We have chosen a rectangular barrier since it has a relatively simple expression. In fact, the approximation in eq.6 (which is a good estimation for any opaque barrier) yields, for all practical purposes, the same results.

In general, eq.4 does not have an analytical solution. Thus, we will begin with the numerical results. In Fig. 2 we present the results of the exact solution of eq.4 for the temporally confined perturbation

$$f(t/\tau)=(f_0/\tau)\exp(-|t/\tau|). \tag{7}$$

The x-axis represents the frequency ω (i.e., the frequency of the transmitted particles), and the y-axis represents the logarithm of τ (the perturbation's characteristic time). Within this matrix, the gray levels indicate the logarithm of the transmission coefficient, i.e.,$\ln|a_\omega G^+_\omega(x>L)|$. In the figure, dark spots indicate a high probability of a particle being emitted with the corresponding frequency, or energy (ω).

From the figure we see that, for very large τ (i.e., the adiabatic case), the outgoing frequency is almost identical to the incoming one, Ω. That is, the particles simply experience spectral widening.

When τ decreases (and thus the process becomes more energetic), the spectral width of the outgoing particles increases, as it should by the uncertainty principle. The strange part comes later, when τ reaches the value τ_0 , after which the increase in the spectral width becomes more moderate. This is the point at which the activated particles prefer to be activated to the first resonant energy $\simeq U+(\pi/2L)^2$. These resonant energies *are* barrier-dependent; therefore, one should expect differences in these resonances from one barrier to the next. However, these resonances have nothing to do with the following effect, which is the main issue in this paper.

When τ decreases even further, the spectral width is split into discrete probable frequencies-like a fan. That is, the transmitting (through the barrier) particles are activated to specific preferred frequencies, while *activation to others frequencies is forbidden.*

To explain this behavior, we can use the approximation that $U-\Omega\gg\tau^{-1}$. Since, $U-\Omega$ can be arbitrarily large, this approximation still enables us to discuss relatively rapid changes. Within this temporal regime, only the spectral vicinity of Ω affects the amplitude a_ω. Thus, we can approximate the Green function in eq. 4 according to: $G^+_\omega(0)\to G^+_\Omega(0)$. The important thing, which allows for this exchange, is that when Ω is very far from U, $G^+_\omega(0)\simeq-(2\sqrt{U-\omega})^{-1}$ does not change much within the spectral range (again, recall that this approximation is almost barrier-independent). Hence, with this assumption in hand eq.4 can readily be solved with a Fourier transform:

$$A(t)=\frac{f(t/\tau)\varphi_\Omega(0)\exp(-i\Omega t)}{1-f(t/\tau)G^+_\Omega(0)} \tag{8}$$

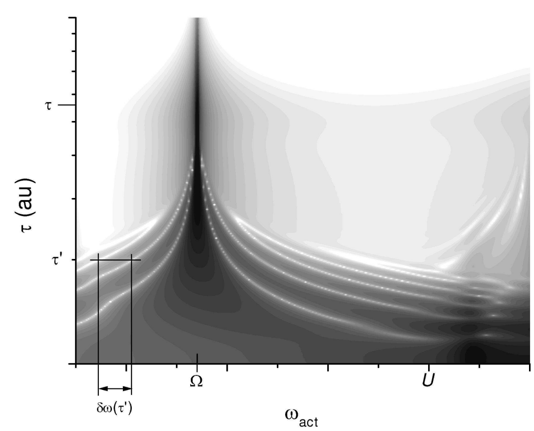

FIG. 2. The probability of a particle being activated to energy ω_{act}, for various characteristic times τ. The y-axis represents τ on a logarithmic scale and the x-axis represents the outgoing activation energy ω_{act}. Inside the figure, the darker the spot the higher the probability it represents (again on a logarithmic scale). For an arbitrary τ', the corresponding gap between adjacent forbidden activation energies, i.e., $\delta\omega(\tau')$ is illustrated.

where $A(t)$ is the inverse Fourier counterpart of a_ω. For convenience, we use the dimensionless function g: $f(t/\tau) \equiv (f_0/\tau)g(t/\tau)$ (where $f_0 \equiv f(0)$). The results in Fig.2, for example, were taken for $g(x) = \exp(-|x|)$. It is assumed that $f_0 \equiv f(0)$ is the maximal value of the perturbation. Therefore,

$$A(t) = \frac{f_0 g(t/\tau)\varphi_\Omega(0)\exp(-i\Omega t)}{\tau - f_0 g(t/\tau) G^+_\Omega(0)} \tag{9}$$

Eq.9 determines two time scales

$$\tau_1 \equiv f_0 G_\Omega \tag{10}$$

and

$$\tau_2 \equiv \tau g^{-1}(-f_0 G_\Omega/\tau). \tag{11}$$

When $\tau \gg \tau_1$, the denominator of eq. 9 can be ignored, and the broadening of the spectral width is exactly equal to the spectral width of the perturbation. Note that since we are dealing with a strong perturbation (large f_0), this characteristic time is very long.

When $\tau < \tau_1$ (and this is the more interesting regime), the denominator cannot be ignored, and here τ_2 comes into play: when $t \ll \tau_2$, $A(t) \simeq \varphi_\Omega(0)\exp(-i\Omega t)/G^+_\Omega(0)$; except for the oscillating term the absolute value of $A(t)$ is almost a constant. However, when $t \gg \tau_2$, the denominator of eq. 9 can again be ignored (because of the rapid decay of g). Therefore, $A(t)$ has a kind of rectangular shape: it is almost a constant for a time $\sim \tau_2 \gg \tau$ and then it abruptly decays with the time scale τ For such a "rectangular" shape, the Fourier transform is something like the sinc function, which is oscillating and decaying at the same time. This oscillating "frequency" of a_ω is τ_2 (note that we are dealing here with the frequency space, hence the frequency has dimensions of time). Thus, one can write $a_\omega \simeq a[(\omega - \Omega)\tau_2]$. Multiplication by the Green function $G^+_\omega(x > L)$ distorts the picture a little (it increases the high frequencies' contributions at the expense of those of the low frequencies), but it does not change the fact that the particles can be activated ("act") only by a product of an integer and the quantity $\delta\omega \sim \tau_2^{-1}$. That is,

$$\omega^n_{act} \simeq \Omega + n\delta\omega, \tag{12}$$

where n is an integer. Similarly, no activation occurs to the forbidden ("for") activation levels

$$\omega^n_{for} \simeq \Omega + \left(n + \frac{1}{2}\right)\delta\omega. \tag{13}$$

These are the forbidden frequencies we were looking for. From the definition of τ_2 (eq. 11), one learns that it depends not only on the characteristic time τ and on the incoming frequency Ω, but also on the strength of the perturbation f_0 and, maybe more surprisingly, on the *functional behavior* of f. For example, suppose

$$g(x) = \exp(-|x|^m) \tag{14}$$

then

$$\tau_2 = \tau\left[\ln\left(-\frac{f_0 G_\Omega}{\tau}\right)\right]^{1/m} \tag{15}$$

It is clear that when the perturbation is strong (f_0 is large) the dependence on m can be very significant.

Let us take the following temporal dependence that corresponds to a milder decay:

$$g_n(x) = \frac{1}{1 + |x|^n} \tag{16}$$

Such temporal behavior leads to a particularly simple solution, which allows tracing the onset of the "discrete activation level effect". In this case

$$a_\omega = \frac{1}{2\pi}\int dt A(t)\exp(i\omega t) = \frac{f_0}{2\pi}\int dx \frac{\exp(ixs)}{|x|^n + \alpha^n} \tag{17}$$

where we adopt the dimensionless parameters $s \equiv (\omega - \Omega)\tau$, $\alpha^n \equiv -G(0)f_0/\tau + 1$, and of course $x \equiv t/\tau$.

But eq.17 can also be rewritten

$$a_\omega = \frac{f_0}{\alpha^{n-1}}\tilde{g}_n(s\alpha) \tag{18}$$

where $\tilde{g}_n$ is the inverse Fourier transform of g_n (see eq.5).

For $y \to \infty$, an asymptotic expression can be written

$$\tilde{g}_n(y) \sim \cos[y\cos(\pi/n)]\exp[-y\sin(\pi/n)], \tag{19}$$

(for $n = 2$ and $n = 4$ this is the only term in the asymptotic expansion).

Thus, the oscillating part of a_ω reads

$$\cos\left[(\omega - \Omega)\tau\left(-\frac{Gf_0}{\tau} + 1\right)^{1/n}\cos\left(\frac{\pi}{n}\right)\right] \tag{20}$$

Similar to the exponential case (eq. 14) we can define a characteristic time

$$\tau_2 \sim \tau(-Gf_0/\tau + 1)^{1/n}\cos(\pi/n) \tag{21}$$

which determines the activation energies according to eq. 12. Similar to eq. 15, $\tau_2 \to \tau$ when the *functional* behavior of the perturbation dies out very quickly ($n \to \infty$). Moreover, eq. 21 also reveals the onset of the discrete activation pattern. The parameter n *must be larger than 2*, otherwise no oscillations occur.

Therefore, we conclude that in order to obtain the discrete activation level effect, the perturbation must die out faster than $(t/\tau)^{-2}$.

It should be noted, that because of the point-like nature of the alternating impurity, the whole derivation of the effect depends on the barrier via the Green function,

however, as approximation (6) suggests, any opaque barrier would yield similar results.

To summarise, the activation energy in the case of a strongly varying potential barrier was investigated. It was shown that when the temporal perturbation amplitude is very large, the particles cannot be activated to certain discrete energies $\omega_{for}^{m} \simeq \Omega + (m+1/2)\delta\omega$, where $\delta\omega \sim [\tau g^{-1}(-f_0 G_\Omega/\tau)]^{-1}$, τ is the characteristic time of the perturbation, f_0 is its strength and g^{-1} is the inverse function of the perturbation's temporal dependence. Moreover, it was demonstrated that in order to see this forbidden activation effect, the perturbation must die out faster than t^{-2} .

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